

Underdetermined mixing matrix estimation by exploiting sparsity of sources

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ABSTRACT

To estimate the mixing matrix in underdetermined mixing systems, we propose a novel method by exploiting the sparsity of sources. We utilize the pairwise relationships among all of the mixture representations to detect the single source points in the time-frequency (TF) domain, *i.e.*, the positions where only one source contributed dominantly. The mixture representations at these single source points are then clustered to estimate the underlying mixing matrix. Since the pairwise relationships among all mixtures are considered in the TF domain, the proposed method can achieve an accurate mixing matrix estimation and be robust in noisy cases. Experimental results indicate that our method is effective in mixing matrix estimation and outperforms five peer methods.

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1. Introduction

Blind source separation (BSS) is a fundamental problem in signal processing and data analysis and has been widely used in different areas, such as biomedical engineering [1], remote sensing [2], and wireless communications systems [3–5]. The aim of BSS is to separate the source signals from the observed mixtures, without any *a priori* knowledge (or with very little knowledge) about the source signals or the mixing process [6,7]. Generally, for the underdetermined case, BSS consists of two stages: the mixing matrix estimation and the source recovery [8,9]. Since the source recovery process is based on the output of the first stage, accurate estimation of the mixing matrix becomes critical for BSS and has attracted increasing interest from those working in the BSS area [10,11]. However, the mixing matrix estimation by using only the mixed output signals is challenging, especially when the sources are more than the observed mixtures, which refers to the underdetermined mixing matrix estimation (UMME) [12].

To estimate the mixing matrix estimation, many algorithms have been developed in recent years. Most of them assume that the source signals are sparse in the time domain or the time-frequency (TF) domain [13–15]. For example, assuming only one source presents at each TF point, Jourjine et al. [16] proposed a

method, called the degenerate unmixing estimation technique (DUET), to estimate the mixing matrix. Nguyen et al. [17] developed a method using the quadratic TF distributions to obtain the mixing matrix. Clearly, the TF-disjoint condition is restrictive. To relax the sparsity constraint, Abrard et al. proposed the time-frequency ratio of mixtures (TIFROM) algorithm [18] by detecting single source areas in time-adjacent windows. In recent years, as some extensions to DUET and TIFROM, Reju et al. [19] proposed a method of detecting the TF points where only single source presents, *i.e.*, single source points (SSPs) [11]. The mixture representations at SSPs are then clustered to estimate the mixing matrix. Specifically, it compares the absolute directions of the real and imaginary parts of the mixture vector at each TF point. Then, it treats the point that having a value smaller than the given threshold as an SSP.

Note that all these methods are based on the detection of SSPs, and their performance depends greatly on the accuracy of the SSPs detection. However, to detect the SSPs, these methods only consider the ratio coefficients of each sample itself or the relationships among adjacent samples. Therefore, they are sensitive to noise in real-world systems and suffer performance degradation in noisy environments [13].

To overcome these problems, we propose a novel method to estimate the mixing matrix. By exploiting the sparsity of source signals in the TF domain, we also aim to detect SSPs to estimate the mixing matrix. Unlike existing methods, our method detects the TF points where one mixture TF vector has some other mixture

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TF vectors be very close to it. We note that some of these TF points are not SSPs, but they nevertheless have a large impact on the estimated results. To address this issue, we propose a new strategy to eliminate these TF points.

Since the pairwise relationships among the representations of mixtures at all TF points are considered, it can obtain a more accurate detection of SSPs. Furthermore, the elimination of the fake SSPs makes our method be able to provide a more accurate estimation result. Different from the existing ones in [17–20], which are based on the ratios of mixing signals, our method considers the whole structure of the observed mixtures, thus being more effective, especially in noisy environments.

2. Background and related work

Without loss of generality, an instantaneous underdetermined mixing system with n sources and m outputs ($n > m$) is defined as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{e}(t), \quad (1)$$

where $\mathbf{s}(t) = [s_1(t), s_2(t), \dots, s_n(t)]^T$, $s_i(t)$ is the coefficient of the i -th source at time instant t , $\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_m(t)]^T$, $x_j(t)$ is the coefficient of the j -th mixture at time instant t , $\mathbf{A} = [\mathbf{a}(1), \mathbf{a}(2), \dots, \mathbf{a}(n)] \in \mathbb{R}^{m \times n}$ is the unknown mixing matrix, *i.e.*, the underlying channels, $\mathbf{e}(t)$ is the noise, and $t = 1, 2, \dots, N$ is the time instant. UMME is to estimate the unknown mixing matrix \mathbf{A} by inputting the observed mixtures $\mathbf{x}(t)$. It has been proved to be challenging, especially for the underdetermined case [21].

Since the source signals would be more sparse in the TF domain, we transform the system in (1) into the TF domain without considering the noises by using a short-time Fourier transform (STFT) [22]:

$$\tilde{\mathbf{x}}(t, k) = \mathbf{A} \tilde{\mathbf{s}}(t, k), \quad (2)$$

where $\tilde{\mathbf{x}}(t, k) = [\tilde{x}_1(t, k), \tilde{x}_2(t, k), \dots, \tilde{x}_m(t, k)]^T$ are the TF representations of the mixtures $\mathbf{x}(t)$, $\tilde{\mathbf{s}}(t, k) = [\tilde{s}_1(t, k), \tilde{s}_2(t, k), \dots, \tilde{s}_n(t, k)]^T$ are the TF representations of sources $\mathbf{s}(t)$, and $\tilde{x}_i(t, k)$ and $\tilde{s}_j(t, k)$ are, respectively, the values of the i -th mixture and the j -th source at the (t, k) TF point.

There are plenty of mixing matrix estimation methods proposed to detect SSPs at first. Then, using the clustering algorithms to classify the mixture vectors at these SSPs into different groups, they compute the center of these groups as the estimated mixing matrix columns. It is clear that performance of these methods depends greatly on the accuracy of the SSPs detection. In the following, we will introduce more details about the single source points detection process of some recently developed mixing matrix estimation methods, *i.e.*, the TIFROM method [18], the method in [19], and the method in [20].

In TIFROM method, it detects adjacent windows where only single source presents. Formally, it calculates the complex ratio between different mixtures at each TF window (for ease of explanation, assume that there are two sources),

$$\alpha(t, k) = \frac{\tilde{x}_1(t, k)}{\tilde{x}_2(t, k)}. \quad (3)$$

The TIFROM method assumes that if only source $s_i(t)$ presents in several time-adjacent windows (t, k) , then $\alpha(t, k)$ should be constant, otherwise its values differ over them. Based on this observation, it computes the sample variance of $\alpha(t, k)$ on series Γ_q of M short half-overlapping time windows corresponding to adjacent t . By applying this procedure to every frequency, it obtains the variance of $\alpha(t, k)$ as

$$\text{var}[\alpha](\Gamma_q, k) = \frac{1}{M} \sum_{t_i \in \Gamma_q} \left(\alpha(t_i, k) - \frac{1}{M} \sum_{t_\mu \in \Gamma_q} \alpha(t_\mu, k) \right)^2. \quad (4)$$

If only single source presents in these M windows, then the variance of $\alpha(t, k)$ equals zero. Otherwise, the variance is different from zero.

In [19], it compares the absolute directions of the real and imaginary parts of the mixture representations and takes the points that having a value less than a given threshold angle as SSPs. Mathematically, it checks the following condition:

$$\left| \frac{\text{real}(\tilde{\mathbf{x}}(t, k))^T \text{imag}(\tilde{\mathbf{x}}(t, k))}{\|\text{real}(\tilde{\mathbf{x}}(t, k))\| \|\text{imag}(\tilde{\mathbf{x}}(t, k))\|} \right| > \cos(\Delta\theta), \quad (5)$$

where $\Delta\theta$ is a given threshold and $\|\cdot\|$ stands for the ℓ_2 -norm. This algorithm has a low computational complexity, thus being potentially useful in online mixing matrix estimation.

In [20], it applies a phase-angle-based detection strategy to distinguish the SPPs at the TF plane. It takes the points where the ratios of the imaginary part and the real part of the mixture TF representations are the same as the SSPs, *i.e.*, the TF points where the following condition holds:

$$\frac{\text{imag}(\tilde{x}_1(t, k))}{\text{real}(\tilde{x}_1(t, k))} = \frac{\text{imag}(\tilde{x}_2(t, k))}{\text{real}(\tilde{x}_2(t, k))}. \quad (6)$$

Considering the actual situation and experience, an error parameter ε is used to relax the restrictions. The condition is given by

$$\left| \frac{\text{imag}(\tilde{x}_1(t, k))}{\text{real}(\tilde{x}_1(t, k))} - \frac{\text{imag}(\tilde{x}_2(t, k))}{\text{real}(\tilde{x}_2(t, k))} \right| < \varepsilon. \quad (7)$$

It is clear that the ratios are sensitive to noises, which makes these methods result in a performance degradation in noisy environments.

3. Proposed method

In this section, we present our method based on the following two assumptions:

Assumption 1. For each source, there exist a number of TF points that only the target source possesses dominant energy.

Assumption 2. All mixing matrix columns have different absolute directions.

Assumption 1 provides for the sparsity of the sources. It is much relaxed in comparison with the assumptions of the method in [17] and the TIFROM [18] algorithm. **Assumption 2** is necessary for distinguishing different source signals and is also used in many methods [19,6], and we can meet it with probability one for randomly generated $m \times n$ matrices.

In the first step, we use STFT to compute the mixture representations in the TF domain and denote them as $\tilde{\mathbf{x}}(t, k)$. Since the elements of $\tilde{\mathbf{x}}(t, k)$ are complex numbers, we set each element as its complex magnitude with the sign of its real part:

$$\tilde{x}_i(t, k) \leftarrow \text{sign}(\text{real}(\tilde{x}_i(t, k))) |\tilde{x}_i(t, k)|. \quad (8)$$

To reduce the impact of the noises, we eliminate the low-energy mixture TF vectors, since the direction of these mixture vectors can be easily affected by noise. Specifically, we eliminate the mixture vectors whose magnitudes are less than ρ times the maximal mag-

nitude of all mixture TF vectors, and ρ is typically set as 0.01. Thus, we eliminate the mixture TF vectors if they meet the following condition:

$$\|\tilde{\mathbf{x}}(t, k)\| < \rho \cdot \max\{\|\tilde{\mathbf{x}}(1, 1)\|, \dots, \|\tilde{\mathbf{x}}(N, K)\|\}, \quad (9)$$

where N and K are the numbers of time instants and frequency bins, respectively, and $\|\cdot\|$ stands for the ℓ_2 -norm.

Next, to clearly illustrate the estimation process, we ensure that all mixture vectors are normalized and their first elements are non-negative by multiplying the mixture vectors whose first elements are negative by minus one. That is, for each mixture representation $\tilde{\mathbf{x}}(t, k)$, we execute the following operation:

$$\tilde{\mathbf{x}}(t, k) \leftarrow \frac{\text{sign}(\tilde{x}_1(t, k)) \tilde{\mathbf{x}}(t, k)}{\|\tilde{\mathbf{x}}(t, k)\|}. \quad (10)$$

From (2), it is easy to find that the single source mixture vectors would have the same or opposite direction with each other. Technically, for any two TF points (t_i, k_j) and (t_μ, k_ν) , the following condition holds:

$$|\tilde{\mathbf{x}}(t_i, k_j)^T \tilde{\mathbf{x}}(t_\mu, k_\nu)| = 1. \quad (11)$$

This inspire us to explore whether the converse is also true; *i.e.*, if condition (5) holds, does it guarantee that the same source occurs at these two TF points?

By analyzing (2), we find that $\tilde{\mathbf{x}}(t_i, k_j)$ and $\tilde{\mathbf{x}}(t_\mu, k_\nu)$ having the same or opposite directions does not guarantee that they are from a single source. There are two cases which lead (5) to be satisfied (for ease of explanation, assume that there are two sources):

Case 1: $\tilde{\mathbf{x}}(t_i, k_j)$ and $\tilde{\mathbf{x}}(t_\mu, k_\nu)$ are from the same source.

Case 2: The energy possessed by the first source has the same ratio as that of the second source at time–frequency point (t_i, k_j) and time–frequency point (t_μ, k_ν) , *i.e.*,

$$\frac{\tilde{s}_1(t_i, k_j)}{\tilde{s}_2(t_i, k_j)} = \frac{\tilde{s}_1(t_\mu, k_\nu)}{\tilde{s}_2(t_\mu, k_\nu)} \quad (12)$$

In practice, the probability of the second case occurring is close to zero, which allows us to transform the single source mixture vector detection problem into finding the mixture vectors that have some other mixture vectors with the same direction or the opposite direction as them.

Under noisy circumstances, we relax the constraint in (11) to detect the SSPs. For each mixture TF vector $\tilde{\mathbf{x}}(t_i, k_j)$, we find some other mixture TF vectors $\tilde{\mathbf{x}}(t_\mu, k_\nu)$ that satisfy the following condition to construct a set:

$$\Omega(t_i, k_j) = \{\tilde{\mathbf{x}}(t_\mu, k_\nu) \mid |\tilde{\mathbf{x}}(t_i, k_j)^T \tilde{\mathbf{x}}(t_\mu, k_\nu)| \geq \sigma\}, \quad (13)$$

where σ is a given threshold, which is determined according to the noisy level, and is typically set as 0.9999. The value of this parameter also can be determined based on preliminary tests on a small representative dataset when it is available by following the idea in [23].

Note that the TF point (t_i, k_j) is not guaranteed to be an SSP when $\Omega(t_i, k_j)$ is not a null set. Actually, $\Omega(t_i, k_j)$ will be non-null at some TF points where the difference of the ratios of the energy possessed by the first source and the second source is small.

To automatically remove these TF points, we propose a new strategy. It takes $\tilde{\mathbf{x}}(t_i, k_j)$ as a single source mixture TF vector if the number of elements in the set $\Omega(t_i, k_j)$ is larger than a threshold counting number η . In other words, it meets the following condition:

$$\#\Omega(t_i, k_j) > \eta, \quad (14)$$

where $\#\{\cdot\}$ denotes the cardinality of the set. The parameter η is interactively determined by the users in the proposed method. It increases gradually from one to the number that leads to the remaining mixture TF vectors being included in only n groups, where $\tilde{\mathbf{x}}(t_i, k_j)$ is connected with $\tilde{\mathbf{x}}(t_\mu, k_\nu)$ only if $|\tilde{\mathbf{x}}(t_i, k_j)^T \tilde{\mathbf{x}}(t_\mu, k_\nu)| \geq \sigma$ and (14) holds. The elimination of the TF points where (14) does not hold is essential for the proposed method.

In this manner, we obtain the mixture TF vectors contributed by a single source. The next step is to cluster these mixture TF vectors into n groups. Finally, we compute the center of each group and take it as the estimated result of one column vector of the mixing matrix. The procedure of our method is summarized in Algorithm 1.

Algorithm 1 The procedure of the proposed underdetermined mixing matrix estimation algorithm

Input: The observed mixtures, the number of sources n , and the value of the hyper parameter σ .

Output: The estimated mixing matrix $\hat{\mathbf{A}}$.

- (1) Obtain the mixture representations in the time–frequency domain by using STFT [22] as Eq. (2).
 - (2) Eliminate the low energy mixture vectors via Eq. (9).
 - (3) Normalize the mixture TF vectors and multiply the vectors whose first elements are negative by minus one as Eq. (10).
 - (4) Select the TF vectors that satisfy Eq. (13).
 - (5) Determine the η in Eq. (14) adaptively and obtain n groups of mixture TF vectors with the k-means algorithm.
 - (6) Calculate the centers of these n groups and take them as the estimation of the mixing matrix $\hat{\mathbf{A}}$.
-

Different from the method in [13], called UBSS-SC, which uses the sparse coding strategy to detect the SSPs, the proposed method adopts the pairwise relationship between all mixture vectors in the TF domain. UBSS-SC performs well and is robust to the noises when the mixing matrix columns are not close to each other. However, it may suffer performance degradation when the mixing matrix columns are close to each other, and it needs to solve a set of ℓ_1 -norm minimisation problems, which results in a high-computational cost in large-scale problems. The proposed method is more robust to the closeness between the mixing matrix columns and has a lower computational complexity since it only needs to calculate the pairwise distances among all mixture vectors. In addition, the proposed method has a step to remove fake single source points, which is critical for improving the estimation accuracy.

4. Simulations

In this section, we evaluate the effectiveness of our method. In the experiments, the setting of the parameters of the proposed method is followed as the recommendation in the previous section.

To begin with, we evaluate the performance of our method¹ in the mixing system with 4 speech sources and 3 mixtures. The mixing matrix is randomly generated, with elements in (0, 1), and given as:

¹ The MATLAB code is available at <https://www.dropbox.com/s/78jxmj5ec128vie/UMME-code.zip?dl=0>.

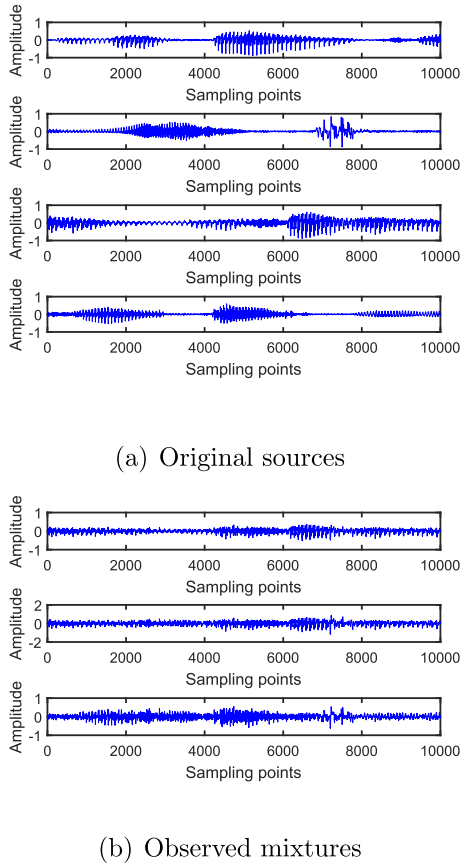


Fig. 1. The waveforms of four voices in (a) and the three mixtures in (b) in the time domain.

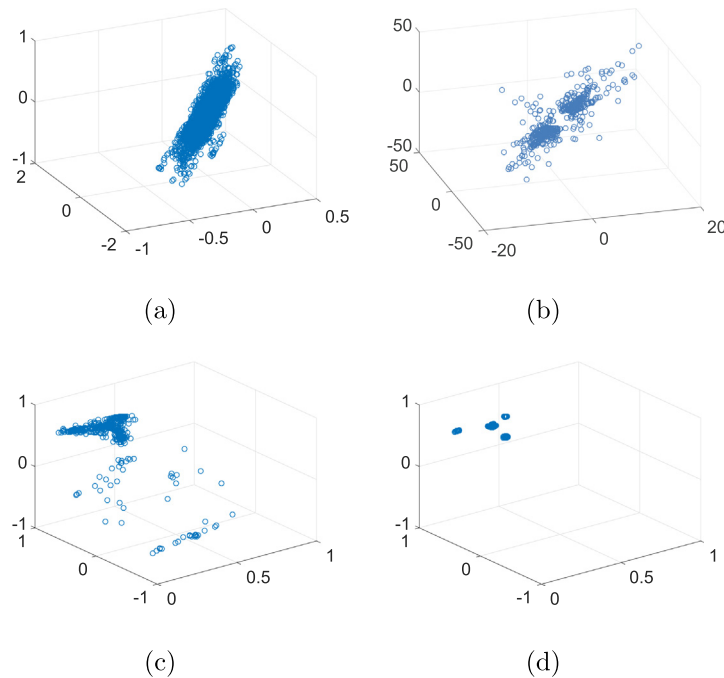


Fig. 2. Scatter plots of the mixtures in different stages: (a) the observed mixtures in time domain; (b) the mixtures in TF domain after eliminating the low-energy TF vectors; (c) the result of the TF vectors after the third step of our method; (d) the TF vectors that satisfied (14) and could be used to estimate the mixing matrix.

$$\mathbf{A} = \begin{pmatrix} 0.2828 & 0.1701 & 0.5517 & 0.3245 \\ 0.5896 & 0.8763 & 0.9674 & 0.4210 \\ 0.3720 & 0.5696 & 0.2284 & 0.8649 \end{pmatrix}, \quad (15)$$

and 10000 samples of four speech source signals, as shown in Fig. 1 (a), are adopted. By mixing the four sources, we obtain the observed three mixtures in Fig. 1(b).

Fig. 2(b) shows the mixture vectors in the TF domain after the elimination of low energy TF vectors. From the result we can see that even though the directions of the observed mixtures are clearer than that in the time domain as shown in Fig. 2(a), they are still mixed unclearly. Also, we can find that there is a hole in side the point cloud due to the elimination of low energy TF vectors. After executing first five steps, we obtain a scatter plot of the TF vectors that satisfy (10) in Fig. 2(c). Since the mixture TF vectors with a single source can be grouped into n clusters, we view the scatter plot result by improving the value of η and obtain the value of η when the mixture TF vectors are grouped into n clusters as shown in Fig. 2(d) by executing step (5) of the proposed method. We can see that the non-single source TF vectors are removed and the obtained groups are close to each other.

Finally, we acquire the estimated mixing matrix with Step (6) of the proposed method and report it in (16). To eliminate the effect of the possible permutations of the mixing matrix columns, we denote the original and final estimation results of the mixing matrix as $\bar{\mathbf{A}}$ and $\hat{\mathbf{A}}$, respectively. We search the estimation result of each mixing matrix column $\mathbf{a}_i, i = 1, 2, \dots, n$, denoted as $\hat{\mathbf{a}}_i$, as the column vector of $\bar{\mathbf{A}}$ which is closest to \mathbf{a}_i . From the results in (16), we can see that the columns of the estimated result $\hat{\mathbf{A}}$ have approximately the same direction as their corresponding columns in the mixing matrix \mathbf{A} . To further investigate our method, we input the result of the proposed method to the source recovery procedure in [13] and obtain the recovered source signals as shown in Fig. 3.

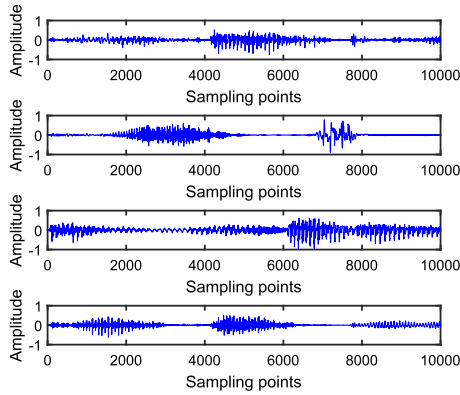


Fig. 3. The waveforms of the recovered sources in the time domain by the proposed method.

$$\hat{\mathbf{A}} = \begin{pmatrix} 0.3761 & 0.1617 & 0.4854 & 0.3212 \\ 0.7837 & 0.8281 & 0.8507 & 0.4200 \\ 0.4942 & 0.5368 & 0.2013 & 0.8487 \end{pmatrix}. \quad (16)$$

From the result, we can see that the sources are well separated. However, some of the source signals were not recovered well since the mixing matrix is not perfectly estimated.

In the second experiment, Monte Carlo runs are used to evaluate the performance of the tested methods under different levels of noises. We compare the proposed method with five other popular methods, *i.e.*, the TIFROM method and the methods in [13,20,19,17]. We use the same mixing system as the previous experiment. Moreover, we examine the robustness of these methods with respect to different levels of the white Gaussian noise added on the mixtures.

To evaluate the performance of the tested methods, we define the performance metric [13]:

$$\text{Error} = \sum_{i=1}^n \left(\frac{\|\mathbf{a}(i) - \delta \hat{\mathbf{a}}(i)\|}{\|\mathbf{a}(i)\| \cdot \|\hat{\mathbf{a}}(i)\|} \right), \quad (17)$$

where $\hat{\mathbf{a}}(i)$ is the estimated result of $\mathbf{a}(i)$ and δ is a scalar for removing the scalar ambiguity [13].

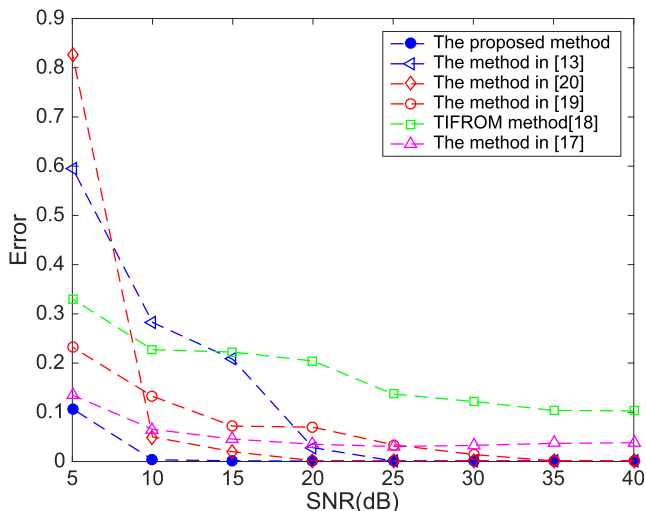


Fig. 4. Performance comparison between the proposed method and the peer methods.

Fig. 4 shows the comparison performance of different methods versus signal-to-noise ratio (SNR) evaluated over 101 Monte Carlo runs, from which we can see that:

- The SSPs detection-based methods, *i.e.*, the methods in [13,20,19] and our method outperform TIFROM and the method in [17] under low-level noise scenarios since there are many TF points with multiple active sources.
- The performance of all evaluated methods decreased with the increase of SNR in the mixed signals, which is consistent with the results of other works [13,20,19].
- The proposed method outperforms the other five algorithms among all noise levels, especially when SNR is less than 20dB, which indicates that our method is robust to noise.
- The methods in [13,20] work well under the noisy level SNR > 20dB. However, they are inferior to other methods under high-level of noise *e.g.*, SNR = 5dB, since they failed to detect real SSPs. Even the methods in [13] is robust to the noise when the mixing matrix columns are not close to each other, it is sensitive to high-level noise in the scenario where the mixing columns are very close to each other.

Table 1 reports the average time cost of these tested methods on a PC (Intel(R) Core(TM) 3.30 GHz, 8 GB RAM) with the MATLAB 2015b platform. We can find that our method is the fastest one. It benefits mainly from the elimination of low-energy TF mixtures and the parallel process of matrix multiplication operation in MATLAB. Since our method considers all the pairwise relation between mixture TF representations, its time cost may sharply increase when number of mixture signals is very large. The method in [17] is the slowest method. It needs a plenty of computational operations to obtain the quadratic TF distributions of the observed mixtures. The time costs of other four methods are a little bit larger than the time cost of the proposed method.

5. Conclusion

This paper focuses on estimating the mixing matrix from their instantaneous mixtures in underdetermined systems. Exploiting the sparsity of sources, we proposed an effect method to blindly estimate the mixing matrix in underdetermined systems. Our method has a more relaxed sparsity constraint on the source signals when compared with some other UMME approaches. Moreover, the pairwise relationships among all samples have been considered and the count number is adaptively determined. The theoretical analysis and experiments on speech sources have demonstrated the effectiveness of our proposed method.

Declaration of Competing Interest

Table 1
The average time cost (ms) of the tested methods.

Method	Time cost (ms)
The proposed method	37.12
The method in [13]	160.21
The method in [20]	76.41
The method in [19]	133.4
TIFROM [18]	43.51
The method in [17]	1540

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

- [1] J. Metsomaa, J. Sarvas, R.J. Ilmoniemi, Blind source separation of event-related EEG/MEG, *IEEE Trans. Biomed. Eng.* 64 (9) (2017) 2054–2064.
- [2] W.K. Ma, J.M. Bioucas-Dias, T.H. Chan, N. Gillis, P. Gader, A.J. Plaza, A. Ambikapathi, C.Y. Chi, A signal processing perspective on hyperspectral unmixing: insights from remote sensing, *IEEE Signal Process. Mag.* 31 (1) (2014) 67–81.
- [3] A. Cichocki, S. Amari, *Adaptive blind signal and image processing: learning algorithms and applications*, vol. 1, John Wiley & Sons, 2002.
- [4] T. Adali, C. Jutten, A. Yeredor, A. Cichocki, E. Moreau, Source separation and applications, *Signal Process. Mag.*, IEEE 31 (3) (2014) 16–17.
- [5] L. Zou, X. Chen, X. Ji, Z.J. Wang, Underdetermined joint blind source separation of multiple datasets, *IEEE Access* 5 (2017) 7474–7487.
- [6] D. Peng, Y. Xiang, Underdetermined blind source separation based on relaxed sparsity condition of sources, *IEEE Trans. Signal Process.* 57 (2) (2009) 809–814.
- [7] G. Chabriel, M. Kleinstueber, E. Moreau, H. Shen, P. Tichavsky, A. Yeredor, Joint matrices decompositions and blind source separation: a survey of methods, identification, and applications, *Signal Process. Mag.*, IEEE 31 (3) (2014) 34–43.
- [8] Y. Luo, W. Wang, J.A. Chambers, S. Lambotharan, I. Proudler, Exploitation of source nonstationarity in underdetermined blind source separation with advanced clustering techniques, *Signal Process.*, IEEE Trans. 54 (6) (2006) 2198–2212.
- [9] P. Chen, D. Peng, L. Zhen, Y. Luo, Y. Xiang, Underdetermined blind separation by combining sparsity and independence of sources, *IEEE Access* 5 (2017) 21731–21742.
- [10] J. Xu, X. Yu, D. Hu, L. Zhang, A fast mixing matrix estimation method in the wavelet domain, *Signal Processing* 95 (2014) 58–66.
- [11] T. Dong, Y. Lei, J. Yang, An algorithm for underdetermined mixing matrix estimation, *Neurocomputing* 104 (2013) 26–34.
- [12] C. Llerena-Aguilar, R. Gil-Pita, M. Utrilla-Manso, M. Rosa-Zurera, A new mixing matrix estimation method based on the geometrical analysis of the sound separation problem, *Signal Processing* 134 (2017) 166–173.
- [13] L. Zhen, D. Peng, Z. Yi, Y. Xiang, P. Chen, Underdetermined blind source separation using sparse coding, *IEEE Trans. Neural Networks Learn. Syst.* 28 (12) (2017) 3102–3108.
- [14] J.J. Thiagarajan, K. Natesan Ramamurthy, A. Spanias, Mixing matrix estimation using discriminative clustering for blind source separation, *Digital Signal Process.* 23 (1) (2013) 9–18.
- [15] J. Liu, X. Wu, W.J. Emery, L. Zhang, C. Li, K. Ma, Direction-of-arrival estimation and sensor array error calibration based on blind signal separation, *IEEE Signal Process. Lett.* 24 (1) (2017) 7–11.
- [16] A. Jourjine, S. Rickard, O. Yilmaz, Blind separation of disjoint orthogonal signals: Demixing n sources from 2 mixtures, in: *Acoustics, Speech, and Signal Processing, 2000. ICASSP'00. Proceedings, 2000 IEEE International Conference on*, vol. 5, IEEE, 2000, pp. 2985–2988.
- [17] N. Linh-Trung, A. Belouchrani, K. Abed-Meraim, B. Boashash, Separating more sources than sensors using time-frequency distributions, *EURASIP J. Appl. Signal Process.* 2005 (2005) 2828–2847.
- [18] F. Abrard, Y. Deville, A time-frequency blind signal separation method applicable to underdetermined mixtures of dependent sources, *Signal Processing* 85 (7) (2005) 1389–1403.
- [19] V.G. Reju, S.N. Koh, Y. Soon, An algorithm for mixing matrix estimation in instantaneous blind source separation, *Signal Processing* 89 (9) (2009) 1762–1773.
- [20] J. Sun, Y. Li, J. Wen, S. Yan, Novel mixing matrix estimation approach in underdetermined blind source separation, *Neurocomputing* 173 (Part 3) (2016) 623–632.
- [21] Y. Li, W. Nie, F. Ye, Y. Lin, A mixing matrix estimation algorithm for underdetermined blind source separation, *Circuits Systems Signal Process.* 35 (9) (2016) 3367–3379.
- [22] L. Cohen, *Time-frequency analysis*, vol. 778, Prentice Hall PTR Englewood Cliffs, NJ, 1995.
- [23] S.M. Aziz-Sbaï, A. Aïssa-El-Bey, D. Pastor, Contribution of statistical tests to sparseness-based blind source separation, *EURASIP J. Adv. Signal Process.* 2012 (1) (2012) 169.